

11th VLTI School of Interferometry

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Practice session on JMMC Tools Interferometry basics

The aim of the following series of exercises is to show you that it is actually quite easy to deal with interferometric data and to teach you the basics about interferometric observations.

The first series of exercises presented here aim at training your practical comprehension of the link between the image space (where you usually work) and the Fourier space (where an interferometer produces its measurements).

This practice session is meant to be carried out with the ASPRO2 software, developed by the Jean-Marie Mariotti Center (JMMC).

1 Getting started with ASPRO2

ASPRO2 is a preparation software for real interferometric observations. However, an imaginary interferometer consisting of 17 North-South-aligned telescopes located at the North Pole has been added for this practice session. This allows us to obtain unrealistic yet interesting UV coverage : aligned strip of regularly-spaced baselines ranging from 1 to 100 m. This strip of baselines can be easily rotated by changing the observational hour-angle.

Now it's time to play.

First launch ASPRO2 and load the first example file : *Example1.asprox* (in /data/ASPRO1). This loads an observation configuration of putative star located at $(0^\circ, +89^\circ)$ observed with our polar interferometer at the autumn equinox during the meridian passage. If you have any problem during the session note that the manual of ASPRO2 is obtained using its HELP button.

Look at the ASPRO2 interface. It is divided into two main parts. The top part has four regions:

- **Targets** : to put either the name of your targets resolved by CDS or their coordinates
- **Main Setting** : to select an interferometer and an instrument, here our fake North-pole "DEMO" interferometer and its "DEMO_SPATIAL" instrument
- **Configuration** : to select a specific baseline or a set of baselines (depending on the interferometer/instrument possibilities)
- **Constraints** : to select the observation date and the minimum elevation for the interferometer

The bottom part is divided into five tabs giving access to :

- **Notebook** : a simple text editor to put comments on the observing programme you prepared with ASPRO
- **Map** : the position of the telescopes of our interferometer, here our North-Pole 17-telescopes interferometer and 136 baselines (that take most of the plot to list)
- **Observability** : the observability of the targets considering the current selected date and interferometer.
- **UV Coverage** : the UV-coverage of our current configuration. Some instrument parameters such as the observing wavelength or mode can be modified here too.
- **OIFits viewer** : plots of the simulated dataset, only available if you have previously selected a model for your target.

In this first practice session, we will mainly work with the last tab **OIFits viewer** and use the **Target**

editor menu to change the model of the object. Contrary to real observations where the amplitude and phase of the complex visibility is not directly measured by optical interferometers, we will mainly look at those values: VISAMP and VISPHI.

— Interactive live demo —

2 Our first model : the uniform disk

It is now time to play with a first model. The one already loaded in the *Example1.asprox* file is a uniform disk. It is the grounding of almost all interferometric-data analysis, and it is very often used, not only to perform stellar diameters fits, but also first-order interpretations of any extended objects.

Question : What is the visibility function of a uniform disk?

Question : Give the relation between the baseline length (B), wavelength (λ), and angular diameter (θ) at the first zero of visibility

This corresponds to the interferometer spatial (or angular) resolution, i.e. the smallest object that can be fully resolved.

Question : Give the same expression with the stellar diameter and wavelength expressed in mas and μm , respectively.

Now look at the **OIFits viewer** tab. Plot the visibility amplitude and phase (VISAMP and VISPHI) as a function of the baseline length (i.e. RADIUS in the software). To do so click on the three dot (...) button at the bottom-right corner, and then select the good X and Y Axes. Roughly measure on the plot the smallest baseline for which the visibility becomes zero and assume that the observations were made in the B band (i.e., at $\lambda=0.46\mu\text{m}$).

Question : What is our object diameter in mas?

Imagine that you obtained a single visibility measurement (i.e. for a single baseline) and assume that the studied object looks like a uniform disk.

Question : In which case(s) can you determine an unambiguous diameter?

Now look at the phase. It is always equal to 0° or 180° . The jump between these two values happens when the visibility amplitude is equal to zero.

Question : Explain this phase signal.

Question : Finally, note the baseline length corresponding to the first zero of visibility and the amplitude of the second-lobe of the visibility function?

3 A few other 1D distributions

Even if the uniform disk model is widely use both to measure stellar diameters and to estimate other objects' extension, it is not the most adapted model for many astrophysical objects.

In this section, we present three commonly used 1D models :

- the **limb darkened disk**, used for accurate stellar diameter measurements.
- the **Gaussian disk**, mostly used for circumstellar disk, stellar wind, and other astrophysical object with no sharp edge.
- the **ring**, adapted to measure inner rim of dusty circumstellar disks, dust shells, or spherically symmetric nebulae.

3.1 Limb darkened disk

Stars are never a uniform disk. They have at least some degree of limb-darkening. Let's replace our uniform disk model with a limb-darkened one. Although this is pretty unrealistic, it is not unusual to express the limb-darkening shape as a quadratic function with coefficients a_1 and a_2 ¹.

Let's see how to modify a target model. First, in the **Edit** menu, click on **Target Editor**. It opens a new window named "Target Editor" that contains three tabs :

- **Targets**, that shows the list of targets and their parameters,
- **Models**, that is used to associate a model (intensity distribution) to a target,
- **Groups** that allows targets of an observing run to be aggregated into groups (for instance as calibrators, primary targets, backup targets...)

In this practice session we are only interested by the second tab. In the **Models** tab, there is only one target, i.e. our hypothetical ($0^\circ, +89^\circ$) star, and one model associated to this target, i.e., *disk1*. The model is briefly described in **Model Description**, and the list of model parameters and their values are given in **Model Parameters**. You can check that the diameter you found in the previous section is close to the one given in this window.

Note that all models have three additional parameters, i.e. *flux_weight*, which is useful only for multi-components models, and the x and y positions, to allow adding off-center components. For our one component models, the flux weight will always be 1 and the position (0,0).

Now, remove the disk model by selecting it in the **Model Description** field and clicking on the **Remove** button, select "**limb_quadratic**" in **model type**, and click on **Add**. Look at the new set of parameters shown in the bottom part of the window. This model has two additional parameters, the limb darkening coefficients *a1_coeff* and *a2_coeff*. You can read their definition in **Model description**. Reasonable values for limb darkening in the visible are $a_1 = 0.9$ and $a_2 = -0.2$. Keep a 4 mas diameter.

Question : Note the baseline length corresponding to the first zero of visibility and the second-lobe amplitude.

Now imagine you would estimate the size of this 4mas limb-darkened star by measuring the position of the first zero of visibility and using the formula determined for a uniform disk in the previous section.

Question : Does the star appears bigger or small than its real size? Explain why.

Question : How can you discriminate between the uniform disk and the limb darkened disk?

3.2 Gaussian distribution

Change the model to a "gaussian" (remove previous), and click on Add. Look at the new set of parameters shown in the bottom part of the window. This model has also four parameters, i.e. *flux_weight1*, $x1$, $y1$ like for all models, and *fwhm1*, the full width at half maximum of the Gaussian distribution. Let's set this value to 4 mas and then click on **Ok** to close the window.

Question : What is the Fourier transform of a Gaussian distribution?

Question : Compare the phase signal to that of the uniform disk.

Question : What is the baseline length corresponding to a visibility of 0?

¹Avoid if possible using these simple parametric limb-darkening functions in real life. It is always best to use the real thing: the image of the star produced by a stellar model: MARCS, SATLAS, etc.

3.3 Ring

Open the **Target editor** again, **Remove** the Gaussian model and **Add** a Ring model instead. This time the model has 5 parameters : $flux_weight1$, $x1$, $y1$, a diameter like the uniform disk, and also a width. Let's consider the case of a 4 mas infinitely-thin ring : $diameter1=4$, and $width1=0$.

Question : What are the baseline length corresponding to the first zero of visibility and the amplitude of the second lobe?

3.4 Comparison of these models

In the previous sections you have build four models (uniform disk, Gaussian distribution, ring, and limb darkened disk) with a characteristic size of 4 mas. Their main difference is the smoothness/sharpness of their intensity distribution.

Question : First, classify the models in term of sharpness/smoothness

Question : What is the relation between the distribution sharpness and the 2nd lobe amplitude ?

3.5 Model confusion

If the largest spatial frequency probed by the interferometer is not of the order of the typical size of your source, this may cause problems when you try to interpret your data.

Question : Compare the visibility obtained at 100m for a 0.9 mas uniform disk and a 0.5 mas Gaussian distribution.

Now switch to the K band using the **UV-Coverage** tab and changing the **Instrument mode**.

Question : Do the same comparison.

Typical uncertainties on the visibility measurements are of the order of a few percent.

Question : In which band(s) can you discriminate between these models and determine the object size?

Go back to the B band, and compare the visibility at 100m from the 0.9 mas uniform disk and a 0.61 mas Gaussian distribution.

Question : How can we discriminate between these two intensity distributions?

3.6 Point source and flat field

There is two additional intensity distributions that are widely used for modelling :

- the **point source** used to model a source too small to be resolved whatever by the interferometer
- the **flat field** used to represent the exact opposite, a fully resolved object.

Note that these models are useless on their own and are only used to build multi-component models described further down.

Question : What are the point source and flat field visibility functions?

In ASPRO2 point sources are modelled using the *punct* function. Flat field are not included, but can be modelled easily using a very extended uniform disk, for example with $D \geq 250$ mas.

4 Going 2D with flattened models

In the previous sections you learned about 1D functions to model interferometric measurements. Now it is time to take a look at two-dimensional intensity distributions.

4.1 Elliptical distributions

Elliptical distributions are among the most simple, yet useful, 2D intensity distributions. They are defined by the same parameters as their circular counterparts, but with two additional parameters:

- the elongation (*elong_ratio*) defined by the ratio of the major axis over the minor axis.
- the major-axis position angle (*major_axis_pos_angle*)

Ellipse (*elong_disk*), elliptical Gaussian (*elong_gaussian*), and elliptical ring (*elong_ring*) are currently included in ASPRO2.

Note that flattened (instead of elongated) models are also included in the software. They are basically the same models but defined by the minor-axis position angle instead of that of the major-axis.

Choose a North-South *elong_disk* model with a 2 mas minor-axis and an elongation ratio of 2.

Remember that we use a set of North-South projected baselines. You can verify it on the **UV coverage** tab. To clearly see the baselines orientation you first need to uncheck the **Plot rise/set uv track** checkbox.

Question : Compare its visibility function to the one obtained for the 4 mas uniform disk.

Now modify the model so that the major-axis is oriented East-West.

Question : Where is the first zero of the visibility function?

Question : What uniform disk diameter does this correspond to?

Finally, put the disk major-axis at 45° .

Question : Can you conclude on the extension measured by an interferometer?

Now, let's use a new set of baselines. Load the configuration file *Example2.asprox*. Look at the UV plan in the **UV coverage** tab. The baselines in the new set all have the same length, i.e. 42m, but they have different position angles, and cover all directions.

Go to the **OIFits viewer** tab and plot the Visibility (VISAMP and VISPHI) as a function of the position angle (POS_ANGLE).

Question : Without looking at the model can you determine the major-axis position angle?

Question : Assuming an elliptical Gaussian model what are the major and minor axes FWHMs?

4.2 Application to geometrically thin disk

Elliptical intensity distributions are widely used to model geometrically-thin circumstellar disks. In this case the flattening is due to the projection of the disk on the sky-plane, i.e., perpendicular to the observer line of sight. At an inclination angle of 0° , i.e. pole-on, the disk is not flattened, and the elongation ratio (i.e. major-axis/minor-axis) grows with the inclination angle.

Question : Find the formula of the elongation ratio for this simple geometrically thin disk model

Question : Conclude on the inclination angle of our object.

5 Composed models

Interesting astrophysical objects are rarely not simple enough to be modelled by a simple geometrical component, and nothing is better than Image Reconstruction, provided there are enough u,v coverage. But model fitting of simple geometrical functions in the visibilities themselves is quasi mandatory to extract sound physical information (fluxes, positions, angles, components spectra...). For instance, to model some circumstellar environment, you often need two components : some elliptical one for the

environment itself, and a uniform disk or point source, for the star if its contribution to the total flux is not negligible.

Imagine that your model is a weighted sum of N components :

$$I_{tot}(x, y) = \sum_{i=1}^N f_i I_i(x, y) \quad (1)$$

where $I_i(x, y)$ is the intensity distribution of the i^{th} component and f_i its relative flux with $\sum f_i = 1$.

Question : What is the visibility function (i.e. normalized Fourier Transform) for this model?

5.1 Star + circumstellar disk model

Let's look at a simple star + circumstellar disk example.

First, load a new observation file : *Example3.aspro*.

Go to the **UV coverage** tab and look at it (again remove the annoying **Plot rise/set uv tracks** option). It is composed two perpendicular strips of baselines : one North-South and one East-West. It will allow us to probe our object along these two perpendicular orientations.

Now, open the **Target Editor** and create a two components model composed of a 0.5 uniform disk (for the star) and an elliptical Gaussian distribution (for the circumstellar disk), with a 8 mas major-axis oriented North-South and a 4 mas minor-axis. Put the flux contribution of each component to 50%.

Plot the visibility amplitude and phase as a function of the baselines lengths.

Question : Describe the visibility function.

Change the flux ratio between the two components and look at the visibility function.

Question : What has been modified? Can you explain why?

Now imagine that you want to constrain the circumstellar environment extension and flattening.

Question : What baseline lengths and orientations will you choose?

Question : Will these set of baselines give information on the stellar surface?

5.2 Binaries

A second kind of very useful two components model is the binary model. We will use a single strip of North-South aligned baselines to explore this model. So, load the *Example1.aspro* file.

Then, open the **Target Editor** and create a model consisting of two point sources. Set their flux to 0.2 and 0.8. Look at the second component, the x_2 and y_2 parameters have been replaced by *sep2* and *pos_angle2*, i.e. polar coordinates. Choose *sep2*=5 and *pos_angle2*=0. This simulates two unresolved stars separated by 5 mas in the North-South orientation, i.e. aligned with our baselines.

Question : Describe and explain the visibility function.

Write down the amplitude of the sinusoidal modulation. Set the flux ratio to 0.1/0.9 and then to 0.5/0.5.

Question : What is the link between the flux ratio and the amplitude of the binary modulation?

Write down the modulation period in meter, and express it in B/λ units (cycles/rad). Then set the binary separation, i.e. *sep2*, to 10 mas. and do this again.

Question : Give the relation between the binary separation (in rad) and the modulation period (in cycles/rad).

Now change θ_2 to 90° : binary in the East-West orientation.

Question : Describe and explain the visibility function.

Do the same but with θ_2 equal to 30° , 45° , and 60° .

Question : How does this affect the visibility function? Why?

Finally replace the components by uniform disks and try various diameters between 0 and 2 mas, for each component, separately.

Question : What is the effect of resolving one component on the visibility function?

6 Effect of the observed wavelength

6.1 Achromatic objects observed at various wavelengths

The angular frequencies accessible by an interferometer, i.e. its angular resolution, depends on the baseline length but also on the observing wavelength. Imagine that our North-Pole interferometer is able to observe in any photometric band between $0.46\mu\text{m}$ (B) and $12\mu\text{m}$ (N).

Question : Which band will give the highest resolution?

To change the observing wavelength go to the **UV Coverage** tab. You can select a photometric band between B and N in the **Instrument mode** list.

Question : Give the ratio between the resolutions in N and B bands

Let's assume a 0.05 uncertainty on our visibility measurement and three baselines of 50, 70 and 100 m.

Question : In which band should we observe a 1 mas star?

Question : Should we observe a 5 mas star with the same configuration?

Now, let's do a multi-wavelength observation. First load the *Example4.aspro* file. It contains a 3 telescopes (S0-S5-S16) configuration of baselines. Unlike for the other exercises the observing wavelength is not fixed to one band but ranges between $0.1\mu\text{m}$ to $10\mu\text{m}$, the "Wide" (and fake) instrument mode of the UV Coverage panel.

Note that for this observation we use a 1 mas uniform disk model.

First plot the visibility and phase as a function of the baseline length to see the visibility of the three baselines.

Question : Explain why we obtain a large range of values for the visibility of each baseline.

Now plot the visibility as a function of the spatial frequency B/λ (SPATIAL_FREQ). The different colors correspond to different wavelengths from purple for the smallest one ($0.1\mu\text{m}$) to red for the largest one ($10\mu\text{m}$).

Question : For this achromatic model case, conclude on the effect of observing at multiple wavelengths.

6.2 Examples of chromatic objects

Up to now we've been working with simple geometric models. But one might want to use home-made models such as outputs from radiative transfer or other codes. For that purpose ASPRO2 allow us to upload images in fits format and computes the visibility corresponding to this model and the selected interferometer and instrument configuration.

Astrophysical objects make and measure are usually wavelength dependent. For instance, the size of a circumstellar environment usually grows with the wavelength, as larger wavelengths probe colder medium, thus further from the central star.

Question : Assuming that a star and its circumstellar environment emit as black-bodies, can you conclude on their flux ratio dependence on the wavelength? Which component will dominate the visible flux? What about the mid-infrared?

We will now load a chromatic model (i.e. a cube of images at different wavelengths) of a classical Be star. The model was computed with DISCO, a semi-physical modelling tool for hot stars surrounded by a gaseous circumstellar disk. DISCO is available through the AMHRA² service (Analysis and Modeling at High Angular Resolution) of the JMMC, a web interface giving access to various astrophysical models (images and image-cubes) dedicated to the analysis of interferometric data.

You can directly access the AHMRA service through the **Models** tab of ASPRO2 **Target Editor**. However, in our case, we have precomputed a specific DISCO model. Load the *disco_model.fits* file. After loading it, you will see the first image in the model corresponding to the first wavelength, i.e. $0.5\mu\text{m}$. You can change the intensity dynamic from **LINEAR** to **LOGARITHMIC**. Doing that, you clearly see the two components of the model, i.e. the star and an elongated circumstellar disk.

Now click on the **Animate** button to see the model dependence with the observing wavelength from $0.5\mu\text{m}$ to $10\mu\text{m}$. You can change the animation speed. The model is huge with 250 channels, and ASPRO2 may be a bit stressed...

Question : How does the model depend on the wavelength?

Before closing the **Target Editor** let's rotate our model by 90° so that our 3 North-South baselines will make measurements along the major-axis (i.e. the equator) of this object. Now, close the **Target Editor** and go to the **OiFits Viewer** tab.

Plot the visibility and phase as a function of the spatial frequency. For chromatic objects, such plots mix-up the spatial and spectral dependency of the object. To disentangle these aspects you can draw imaginary lines between points of the same colour (i.e. wavelengths) : this will give you information on the object spatial distribution at each wavelength.

Question : Is the object more extended in the visible ($0.5\mu\text{m}$) or the mid-infrared ($10\mu\text{m}$)?

Question : Assuming that the star is a uniform disk, estimate the stellar diameter in mas

Question : Assuming a distance of 60pc give the stellar Radius in R_\odot .

²<https://amhra.oca.eu/>