

formulae cheat-sheet for simple models

1 Generic properties of Fourier Transform

- **linearity (addition):** $FT[f + g] = FT[f] + FT[g]$,
- **translation (shift):** $FT[f(x - x_0, y - y_0)] = FT[f](u, v) \times e^{2i\pi \cdot (ux_0 + vy_0)}$,
- **similarity (zoom and shrink):** $FT[f(\alpha x, \beta y)] = \frac{1}{\alpha\beta} FT[f]\left(\frac{u}{\alpha}, \frac{v}{\beta}\right)$,
- **convolution (“blurring”):** $FT[f \otimes g] = FT[f] \times FT[g]$,
- ∞ **limit (“small” details):** $FT[f] \xrightarrow{\infty} 0$,
- 0 **limit (“large” details):** $FT[f] \xrightarrow{0} 1$.
- **(u,v) symmetry:** if f is a real function, the modulus of $FT[f]$ is an even function and the phase of $FT[f]$ is an odd function.

2 Generic models

Shape	Brightness distribution	Visibility
Point source	$\delta(\vec{x})$	1
Background	I_0	$\begin{cases} 1 & \text{if } \rho = 0 \\ 0 & \text{otherwise} \end{cases}$
Binary star	$I_0 [\delta(\vec{x}) + R\delta(\vec{x} - \vec{x}_0)]$	$\sqrt{\frac{1+R^2+2R \cos\left(\frac{2\pi\vec{\rho}\cdot\vec{x}_0}{\lambda}\right)}{(1+R)^2}}$
Gauss	$I_0 \sqrt{\frac{4 \ln(2\varnothing)}{\pi}} \times e^{-4 \ln 2 \frac{r^2}{\varnothing^2}}$	$e^{-\frac{(\pi\varnothing\rho)^2}{4 \ln 2}}$
Uniform disk	$\begin{cases} \frac{4}{\pi\varnothing^2} & \text{if } r < \frac{\varnothing}{2} \\ 0 & \text{otherwise} \end{cases}$	$\frac{2J_1(\pi\varnothing\rho)}{\pi\varnothing\rho}$
Ring	$\frac{1}{\pi\varnothing} \delta\left(r - \frac{\varnothing}{2}\right)$	$J_0(\pi\varnothing\rho)$
Exponential	$e^{-k_0 r}, k_0 \geq 0$	$\frac{k_0^2}{1+k_0^2\rho^2}$
Any circular object	$I(r)$	$2\pi \int_0^\infty I(r) J_0(2\pi r \rho) r dr$
Pixel (image brick)	$\begin{cases} \frac{1}{lL} & \text{if } x < l \text{ and } y < L \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin(\pi x l) \sin(\pi y L)}{\pi^2 x y l L}$
Limb-darkened disk (linear)	$\begin{cases} I_0 [1 - u_\lambda (1 - \mu)] & \text{if } r < \frac{\varnothing}{2} \\ \mu = \cos(2r/\varnothing) \end{cases}$	$\begin{cases} \left[\frac{\alpha J_1(x) + \beta \sqrt{\pi/2} \frac{J_{3/2}(x)}{x^{3/2}}}{\left(\frac{\alpha}{2} + \frac{\beta}{3}\right)^2} \right]^2 \\ \alpha = 1 - u_\lambda \\ \beta = u_l \lambda \\ x = \pi \theta_{LD} \frac{B}{\lambda} \end{cases}$

\varnothing = either the diameter for a ring or uniform disk, or FWHM. r or \vec{s} represent the angles in the image plane. $\rho = \frac{B}{\lambda}$ or $\vec{\rho}$ are the spatial frequencies.